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INTEGRATING PROBLEM-BASED LEARNING AND DEEP LEARNING TO ENHANCE MATHEMATICAL REASONING AMONG JUNIOR HIGH SCHOOL STUDENTS: A ONE-GROUP PRETEST–PPOSTTEST QUASI-EXPERIMENTAL STUDY

(Research article)

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Abstract

Mathematical reasoning is a fundamental competency in contemporary mathematics education, yet many junior high school students continue to experience difficulties in constructing logical arguments, connecting mathematical concepts, and solving complex problems. This study examined changes in junior high school students' mathematical reasoning following the implementation of Problem-Based Learning (PBL) integrated with a Deep Learning approach. A quantitative one-group pretest–posttest quasi-experimental design was employed involving 30 Grade VIII students from a public junior high school in North Sumatra, Indonesia. The instructional intervention was implemented across four mathematics learning sessions on Systems of Linear Equations in Two Variables to promote meaningful learning, reflective thinking, and active problem solving. Data were collected using a validated mathematical reasoning test administered before and after the intervention. Descriptive statistics, the Kolmogorov–Smirnov normality test, paired-sample t-test, and N-Gain analysis were used. The findings revealed a statistically significant improvement in students' mathematical reasoning following the instructional intervention, with mean scores increasing from 41.73 in the pretest to 79.40 in the posttest ($t(29) = 18.063$, $p < .001$, Cohen's $d = 3.30$). Because the study employed a one-group pretest–posttest design, these findings indicate substantial within-group improvement following the intervention rather than definitive evidence of causal effectiveness. The mean N-Gain score of 0.64 indicated a moderate level of improvement. The results suggest that integrating PBL with a Deep Learning approach is associated with improvements in mathematical reasoning. The study contributes to innovative mathematics instruction and offers practical implications for improving teaching and learning in education.

Keywords: Problem-based learning; deep learning; mathematical reasoning; mathematics education

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1. Introduction

1.1 The importance of mathematical reasoning

Education in the twenty-first century is increasingly expected to develop learners who are capable of thinking critically, solving complex problems, and adapting to rapidly changing social and technological environments. Contemporary educational systems are no longer focused solely on the transmission of knowledge but also on the development of competencies that enable students to apply knowledge meaningfully in diverse contexts (Destini, F., Syafridin, U., & Drupadi, R., 2026). As societies become more interconnected and knowledge-driven, schools are challenged to prepare learners who can make informed decisions, analyse information critically, and respond effectively to real-world challenges. Consequently, educational innovations that support the development of higher-order thinking skills have become a major priority in many educational systems worldwide (Yapatang, L., & fon Polyiem, T., 2022).

Within this context, mathematics education occupies a central role because it provides opportunities for students to develop logical thinking, analytical reasoning, and systematic problem-solving abilities (Yuhana, Y., & Fajari, L. E. W., 2025). Mathematical learning experiences encourage students to identify patterns, formulate conjectures, evaluate evidence, and construct logical arguments. These competencies are essential not only for academic success but also for effective decision-making in everyday life and future professional practice (Yang, S., & Berdine, G., 2021; Muzakkir, M. A., 2021). Accordingly, the development of mathematical reasoning has become one of the primary goals of mathematics education.

Mathematical reasoning refers to the ability to analyse relationships among mathematical concepts, formulate logical arguments, justify procedures, and draw valid conclusions based on evidence and established principles (Putra & Yulanda, 2021; Rohmatulloh et al., 2022). Students who possess strong mathematical reasoning skills are better able to interpret problems, select appropriate strategies, evaluate solutions, and transfer knowledge across different contexts. For this reason, mathematical reasoning is widely recognized as a fundamental component of mathematical proficiency and an essential competency for twenty-first-century learning (OECD, 2019; Redhana, 2019).

1.2. Challenges in developing mathematical reasoning

Despite its recognized importance, numerous studies indicate that students continue to experience difficulties in mathematical reasoning. Many learners demonstrate weaknesses in connecting mathematical concepts, explaining their thinking, and applying knowledge to unfamiliar situations (Treepob, H., Hemtasin, C., & Thongsuk, T., 2023). Learning activities frequently emphasize procedural fluency and formula memorization, while opportunities for conceptual exploration, justification, and reflective thinking remain limited (Shongwe, B., 2024). As a result, students may successfully perform routine calculations but struggle when required to construct arguments, analyse relationships, or solve non-routine problems.

The challenge is particularly evident within the Indonesian educational context. The Merdeka Curriculum emphasizes meaningful learning experiences that promote critical thinking, creativity, and reasoning competencies necessary for addressing contemporary challenges (Kemendikbudristek, 2022). However, evidence from national and international assessments suggests that Indonesian students' mathematical reasoning performance remains below expected levels. Findings from the Programme for International Student Assessment (PISA) indicate that many students experience difficulties in applying mathematical concepts and reasoning effectively when confronted with complex situations (OECD, 2019). Research by Hasanah et al. (2023) further revealed that students often rely heavily on memorization rather than conceptual understanding, limiting their ability to establish logical connections among mathematical ideas. These findings suggest a need for instructional approaches that actively engage learners in meaningful reasoning processes and conceptual knowledge construction.

1.3 Problem-Based Learning in mathematics education

One instructional model that has received considerable attention in mathematics education is Problem-Based Learning (PBL). PBL positions authentic and contextual problems as the starting point of learning and encourages students to investigate, analyse, and develop solutions collaboratively (Nurcahyaning Kusumawardani et al., 2022; Vatillah et al., 2020). Through active engagement in problem-solving activities, students are encouraged to construct knowledge independently, justify their reasoning, and evaluate alternative solutions. Previous studies have reported that PBL contributes positively to student engagement, conceptual understanding, critical thinking, and mathematical reasoning (Sari, Y. I., Sumarmi, Utomo, D. H., & Astina, I. K., 2021). Nevertheless, the implementation of PBL in classroom practice does not always achieve its intended outcomes. In some cases, instructional activities focus primarily on obtaining solutions rather than promoting deeper reflection and conceptual understanding (Sulistiani & Lusiana, 2024).

1.4. Deep Learning as a pedagogical approach

To address this limitation, educational researchers have increasingly emphasized the importance of Deep Learning as an instructional approach that promotes meaningful and enduring understanding. In educational contexts, Deep Learning refers to learning experiences that encourage students to engage cognitively, emotionally, and reflectively with knowledge rather than merely memorizing information (Mutmainnah et al., 2025). Deep Learning emphasizes meaningful learning, mindful learning, and joyful learning, enabling students to understand not only how concepts are applied but also why they are important and how they relate to broader conceptual structures (Feriyanto & Anjariyah, 2024; Polat & Özkaya, 2023). Such learning experiences support conceptual understanding, reflective thinking, sustained cognitive engagement, and long-term knowledge retention. This emphasis on sustained cognitive engagement is consistent with previous research suggesting that maintaining learners' attention and minimizing cognitive fragmentation facilitate deeper learning processes and more meaningful knowledge construction (Bendo, 2026a).

1.5. Integrating Problem-Based Learning and Deep Learning

The integration of PBL and Deep Learning offers a promising instructional framework for developing mathematical reasoning. PBL provides authentic problem-solving experiences that require students to investigate situations, formulate mathematical representations, and justify conclusions, while Deep Learning strengthens these processes through reflection, conceptual connection, and meaningful understanding (Kusuma & Amelia, 2018; Salmi et al., 2023). Through this integration, students are encouraged not only to solve mathematical problems but also to examine the reasoning underlying their solutions and connect mathematical concepts in meaningful ways.

Previous empirical research consistently indicates that both Problem-Based Learning (PBL) and Deep Learning support higher-order learning outcomes, although they have largely been investigated independently. Studies on PBL have demonstrated positive effects on mathematical reasoning, conceptual understanding, critical thinking, and problem-solving ability (Afifah et al., 2020; Fardian & Dasari, 2023). Likewise, studies examining Deep Learning have reported improvements in students' reasoning, conceptual understanding, reflective learning, and confidence in mathematics (Wahyudi, 2025). More recently, emerging evidence suggests that integrating PBL with Deep Learning may strengthen higher-order thinking skills by combining authentic problem solving with meaningful conceptual engagement (Ardhana et al., 2025). However, empirical investigations specifically examining whether this integrated instructional approach enhances junior high school students' mathematical reasoning remain limited.

1.6. Research gap and purpose of the study

Despite the growing interest in both Problem-Based Learning and Deep Learning, empirical research examining their combined effect on junior high school students’ mathematical reasoning ability remains limited, particularly within Indonesian secondary mathematics education settings. Existing studies have generally investigated these approaches separately or focused on outcomes other than mathematical reasoning. Consequently, there remains a need for empirical evidence regarding the effectiveness of integrating Problem-Based Learning and Deep Learning as a unified instructional approach for improving students’ mathematical reasoning ability.

Collectively, the existing literature demonstrates strong evidence supporting the effectiveness of PBL in developing mathematical reasoning and growing evidence that Deep Learning promotes meaningful understanding and reflective thinking. Nevertheless, these bodies of research have evolved largely in parallel rather than as an integrated instructional framework. Most previous studies have evaluated each approach independently, focused on different learning outcomes, or examined higher-order thinking more broadly rather than mathematical reasoning specifically. Consequently, there remains limited empirical evidence regarding whether integrating PBL with Deep Learning provides additional benefits for developing junior high school students' mathematical reasoning.

Addressing this gap, the present study investigates changes in students' mathematical reasoning following the implementation of Problem-Based Learning integrated with a Deep Learning approach in enhancing junior high school students' mathematical reasoning ability at SMP Negeri 1 Sinunukan, Mandailing Natal Regency, North Sumatra, Indonesia. In addition to examining whether the instructional approach significantly influences students' mathematical reasoning ability, the study also evaluates the extent of improvement following the learning intervention. The findings are expected to contribute to the growing body of knowledge on innovative mathematics instruction and provide practical implications for developing more meaningful, reflective, and reasoning-oriented learning environments.

1.7. Research Hypothesis

This study hypothesizes that integrating Problem-Based Learning with Deep Learning significantly improves junior high school students' mathematical reasoning ability. Specifically, the study tests the following hypotheses:

H1. Junior high school students' mathematical reasoning scores differ significantly between the pretest and posttest following the implementation of Problem-Based Learning integrated with a Deep Learning approach.

1.8. Research Questions

This study investigates changes in junior high school students' mathematical reasoning ability following the implementation of Problem-Based Learning integrated with a Deep Learning approach. Specifically, the study addresses the following research questions:

1. Is there a statistically significant difference between students' mathematical reasoning scores before and after the implementation of Problem-Based Learning integrated with a Deep Learning approach?

2. What is the magnitude of students' mathematical reasoning improvement following the instructional intervention, as indicated by the normalized gain (N-Gain) score?

2. Conceptual Framework

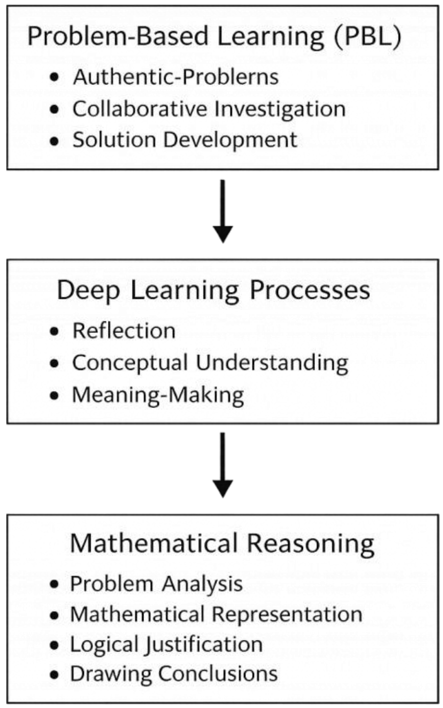

Figure 1. Conceptual framework illustrating the integration of Problem-Based Learning and Deep Learning for enhancing students' mathematical reasoning ability

Figure 1 illustrates the theoretical mechanism underlying the instructional approach examined in this study. Problem-Based Learning provides authentic problem-solving experiences through collaborative investigation, exploration, and solution development. These activities are intentionally strengthened through Deep Learning processes emphasizing reflection, conceptual understanding, and meaning-making. Rather than functioning as separate instructional approaches, Deep Learning processes are embedded throughout the Problem-Based Learning cycle to encourage students to critically examine mathematical ideas, connect new concepts with prior knowledge, evaluate alternative solution strategies, and justify mathematical conclusions. Through this integrated learning process, students are expected to strengthen multiple dimensions of mathematical reasoning, including problem analysis, mathematical representation, logical justification, and the formulation of valid conclusions.

3. Method

3.1. Research design

This study employed a quantitative approach using a one-group pretest-posttest quasi-experimental design to examine changes in students' mathematical reasoning ability following the implementation of Problem-Based Learning (PBL) integrated with a Deep Learning approach. The design involved administering a pretest before the instructional intervention and a posttest after the intervention to evaluate within-group changes in students' mathematical reasoning performance. One-group pretest-posttest designs are widely used in educational research to examine instructional innovations when random assignment or comparison groups are not feasible in authentic classroom settings (Creswell & Creswell, 2018; Ary et al., 2019; Dare & Ring-Whalen, 2024). This design has also been applied in recent educational intervention research to investigate changes in learning outcomes following instructional innovations (Bendo, 2026b). The present design was selected because the intervention was implemented within an existing classroom where random assignment and the establishment of a parallel comparison group were not feasible due to administrative, instructional, and ethical considerations. Nevertheless, because no comparison group was included, alternative explanations for the observed improvements, including testing effects, maturation, classroom experiences, and historical influences, cannot be completely excluded. Accordingly, the findings should be interpreted as evidence of within-group improvement following the instructional intervention rather than definitive evidence that the intervention alone caused the observed changes. Although the design limits the strength of causal inferences, it provides valuable preliminary evidence regarding the feasibility and potential of integrating PBL with a Deep Learning approach in an authentic junior high school mathematics classroom. Future studies employing randomized or controlled quasi-experimental designs are recommended to establish stronger causal inferences. The research design is represented as:

$$\mathbf{O_1 – X – O_2}$$

where $\mathbf{O_1}$ represents the pretest, **X** represents the instructional intervention consisting of Problem-Based Learning integrated with Deep Learning, and $\mathbf{O_2}$ represents the posttest.

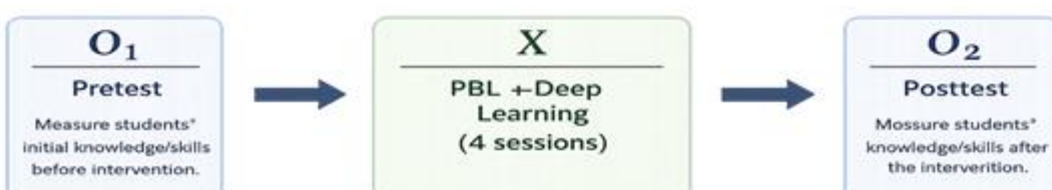

Figure 2. One-group pretest–posttest quasi-experimental research design employed in the study

3.2. Research Context and Participants

The study was conducted at SMP Negeri 1 Sinunukan, Mandailing Natal Regency, North Sumatra, Indonesia, during the 2025/2026 academic year. The school implements the Indonesian Merdeka Curriculum and serves students from diverse socioeconomic backgrounds.

The participants consisted of one Grade VIII-A class selected through purposive sampling. The class was chosen based on its readiness to participate, stability of student enrollment, and suitability of the instructional schedule for implementing the intervention. A total of 30 students participated in the study, with ages ranging from 13 to 15 years. All learning activities were conducted during regular mathematics lessons.

The sample size represented the total enrollment of the selected Grade VIII-A classroom and therefore constituted the entire accessible population for the instructional intervention. Although relatively small, a sample of 30 participants is appropriate for paired-sample t-test analysis and is consistent with classroom-based quasi-experimental research conducted in authentic educational settings (de Winter, 2013). Nevertheless, because the participants were drawn from a single purposively selected classroom, the findings should be interpreted within the context of the study and should not be generalized beyond similar educational settings.

Purposive sampling is commonly employed in classroom-based educational research where specific participant characteristics are required to address the research objectives effectively (Cohen et al., 2018; Etikan & Bala, 2017; Campbell et al., 2020).

3.3. Instrumentation

Data were collected using a mathematical reasoning test consisting of 25 four-option multiple-choice items developed to assess students' ability to understand mathematical problems, formulate mathematical models, apply systematic solution procedures, provide logical justifications, and draw valid conclusions within the topic of Systems of Linear Equations in Two Variables (SPLDV). Scores were computed as the total number of correct responses.

Although the instrument employed a multiple-choice format, the items were specifically designed to measure mathematical reasoning rather than factual recall or procedural computation. Students were required to analyze contextual mathematical situations, identify relevant relationships, formulate mathematical models, evaluate alternative solution strategies, justify mathematical arguments, and select conclusions supported by mathematical evidence. The distractors were constructed to distinguish sound reasoning from common misconceptions, thereby assessing higher-order cognitive processes including analysis, evaluation, and justification (Haladyna & Rodriguez, 2013; Brookhart, 2010). Prior to the main study, the instrument underwent content validation by three doctoral-level experts in psychology, comprising two experts from the Philippines and one from Indonesia. The experts evaluated each item for content relevance, clarity, language appropriateness, and alignment with the intended mathematical reasoning indicators, and their recommendations were incorporated into the revised instrument. A pilot study was subsequently conducted with students possessing characteristics comparable to the target population but who were excluded from the main study. Pilot testing examined item validity, internal consistency reliability, item difficulty, and item discrimination. The instrument demonstrated good internal consistency (Cronbach's α = .82), and only items satisfying the predetermined psychometric criteria were retained for the final version, supporting its suitability for assessing junior high school students' mathematical reasoning (DeVellis & Thorpe, 2021; Hair et al., 2022; Taherdoost, 2023).

The final instrument comprised 25 items distributed across six mathematical reasoning indicators: Understanding Problems (3 items), Formulating Mathematical Models (3 items), Conducting Systematic Solution Procedures (4 items), Providing Logical Justifications (4 items), Drawing Valid Conclusions (4 items), and Combined Mathematical Reasoning (7 items), providing balanced coverage of the multidimensional construct of mathematical reasoning.

3.4. Instructional Intervention

The instructional intervention consisted of four mathematics learning sessions integrating Problem-Based Learning (PBL) with Deep Learning principles and focused on the topic of Systems of Linear Equations in Two Variables (SPLDV). Learning activities were structured around authentic and contextual mathematical problems that required students to analyse situations, identify relevant information, formulate mathematical representations, and determine appropriate solution strategies.

Table 1. Sequence of the Four-Session Problem-Based Learning Integrated with Deep Learning Instructional Intervention

Session	**PBL Stage**	**Deep Learning Process**	**Learning Activities**	**Targeted Mathematical Reasoning Skill**
1	Problem Orientation	Meaning-making	Students explored an authentic SPLDV problem, identified known and unknown information, and discussed possible solution approaches.	Identifying relevant mathematical information and analysing problems
2	Investigation	Conceptual Understanding	Students worked collaboratively to formulate mathematical models, investigate solution strategies, and connect concepts with prior knowledge.	Constructing mathematical representations
3	Solution Development and Presentation	Reflection	Students presented solutions, justified mathematical procedures, evaluated alternative strategies, and received peer feedback.	Providing logical justification
4	Reflection and Evaluation	Knowledge Transfer and Metacognitive Reflection	Students reflected on learning, revised solutions, generalized mathematical concepts, and applied them to new contextual problems.	Drawing valid conclusions and transferring knowledge

Table 2. Alignment of Problem-Based Learning Stages, Deep Learning Processes, and Targeted Mathematical Reasoning Skill

Problem-Based Learning Phase	Deep Learning Process	Mathematical Reasoning Developed
Problem Orientation	Meaning-making	Problem analysis
Investigation	Conceptual understanding	Mathematical representation
Solution Development	Reflection	Logical justification
Evaluation	Knowledge transfer and metacognitive reflection	Drawing valid conclusions

As summarized in Tables 1 and 2, Problem-Based Learning stages were systematically integrated with corresponding Deep Learning processes throughout the four instructional sessions. Each learning activity was intentionally designed to promote progressively deeper conceptual understanding and mathematical reasoning through authentic problem solving, collaborative investigation, structured reflection, and knowledge transfer. Throughout the intervention, students analysed contextual SPLDV problems, constructed mathematical representations, justified solution strategies, evaluated alternative approaches, and reflected on their learning. Rather than emphasizing procedural accuracy alone, the instructional approach encouraged students to connect new concepts with prior knowledge, explain the reasoning underlying their solutions, and draw evidence-based mathematical conclusions. The integration of Problem-Based Learning and Deep Learning was therefore intended to foster meaningful conceptual understanding while strengthening students' mathematical reasoning ability (Savery, 2015; Feriyanto & Anjariyah, 2024; Martaningsih et al., 2022).

3.5. Data Collection Procedure

Data collection was conducted in three sequential phases. Initially, participants completed a pretest to establish baseline levels of mathematical reasoning ability. This was followed by the implementation of the instructional intervention across four consecutive learning sessions employing Problem-Based Learning integrated with Deep Learning principles. Upon completion of the intervention, students undertook a posttest equivalent in content coverage and difficulty level to the pretest. All assessments were administered under standardized classroom conditions and supervised by both the researcher and classroom teacher to ensure consistency in testing procedures and data collection practices (Creswell & Creswell, 2018; Fraenkel et al., 2019).

The pretest was administered immediately before the first instructional session at the beginning of October 2025 to establish students' baseline mathematical reasoning ability. The instructional intervention was then implemented across four consecutive learning sessions during the same month, after which the posttest was administered immediately following the final instructional session. The same validated mathematical reasoning instrument was administered at both measurement occasions to ensure consistency in assessing the targeted learning outcomes. Although repeated administration of the same instrument may have introduced testing effects, this approach is commonly employed in one-group pretest-posttest educational research to facilitate direct comparison of students' performance over time. Accordingly, the findings should be interpreted with appropriate consideration of this methodological limitation.

3.6. Data Analysis

Data were analysed using both descriptive and inferential statistical techniques. Descriptive statistics, including minimum scores, maximum scores, means, and standard deviations, were computed to summarize students’ mathematical reasoning performance before and after the intervention. Test scores were converted to a percentage scale ranging from 0 to 100 prior to statistical analysis. Prior to hypothesis testing, the Kolmogorov–Smirnov test was performed to examine the normality of score distributions. Because the data satisfied the assumption of normality, a paired-sample t-test was employed to determine whether significant differences existed between pretest and posttest scores. To evaluate the magnitude of learning improvement, normalized gain (N-Gain) analysis was calculated following Hake (1999). The combined use of paired-sample t-tests and N-Gain analysis provided evidence regarding both the statistical significance and practical magnitude of the instructional intervention’s effect on students’ mathematical reasoning ability (Field, 2024; Pallant, 2020; Hake, 1999).

The normalized gain (N-Gain) was calculated using the formula proposed by Hake (1998), which estimates the proportion of possible improvement achieved by students between the pretest and posttest. According to Hake's (1998) classification, N-Gain values below 0.30 indicate low improvement, values between 0.30 and 0.69 indicate moderate improvement, and values of 0.70 or higher indicate high improvement. In the present study, N-Gain was used solely as a descriptive indicator of students' learning improvement and not as independent evidence of instructional effectiveness.

3.7. Ethical Considerations

This study was conducted in accordance with established ethical principles for educational research involving human participants. Prior to data collection, permission to conduct the study was obtained from the administration of SMP Negeri 1 Sinunukan, Mandailing Natal Regency, North Sumatra, Indonesia. Because the participants were junior high school students aged 13–15 years, written informed consent was obtained from parents or legal guardians, while student assent was secured before participation. Participants were informed of the study’s purpose, the voluntary nature of their involvement, and their right to withdraw at any time without penalty. To protect privacy and confidentiality, no personally identifiable information was collected or reported. All responses were anonymized, used solely for research purposes, and reported in aggregate form. These procedures were implemented to safeguard participant welfare and ensure compliance with recognized ethical standards for educational research (American Educational Research Association [AERA], 2011; British Educational Research Association [BERA], 2024; Israel & Hay, 2023).

4. Results

The effectiveness of Problem-Based Learning integrated with Deep Learning was examined by comparing students' pretest and posttest mathematical reasoning scores. Descriptive and inferential statistical analyses were conducted to evaluate changes in students' mathematical reasoning ability following the instructional intervention. The following section presents the descriptive and inferential statistical results comparing students' mathematical reasoning scores before and after the instructional intervention. Interpretations regarding the educational significance of these findings are presented in the Discussion section.

Table 3. Mathematical Reasoning Ability

Statistic	**Value**
Number of Students (*n*)	30
Minimum Score	20
Maximum Score	68
Mean (M)	41.73
Standard Deviation (SD)	12.39

Table 3 presents the pretest results of students' mathematical reasoning ability. The mean score of 41.73 indicates relatively low initial reasoning performance. The wide score range (20–68) and standard deviation (12.387) suggest variation in students' prior understanding and reasoning skills. These results establish the baseline for evaluating the effectiveness of the instructional intervention.

Table 4. Normality of Pretest Scores

Data	**Statistic (W)**	**df**	**Sig. (p)**	**Conclusion**
Pretest	0.131	30	0.200	Normally distributed

The Kolmogorov–Smirnov test indicated that the pretest scores were normally distributed ($p = .200 > .05$), satisfying the assumptions for parametric analysis. The normal distribution of scores suggests that students' initial mathematical reasoning abilities were sufficiently representative of the sample and appropriate for further statistical examination.

Table 5**.** Mathematical Reasoning Ability After the Intervention

Statistic	**Value**
Number of Students (*n*)	30
Minimum Score	62
Maximum Score	96
Mean (M)	79.40
Standard Deviation (SD)	9.87

Table 5 demonstrates a substantial improvement in students' mathematical reasoning following the instructional intervention. The mean score increased from 41.73 in the pretest to 79.40 in the posttest, representing an average gain of more than 37 points. Improvements were also observed across the descriptive statistics, with the minimum score increasing from 20 to 62 and the maximum score increasing from 68 to 96. The posttest standard deviation (SD = 9.873) was lower than the pretest standard deviation (SD = 12.387), indicating that posttest scores were less dispersed than pretest scores. However, this descriptive finding should not be interpreted as evidence that all students improved similarly because individual learning trajectories were not examined. Overall, the descriptive statistics indicate higher mathematical reasoning performance following the instructional intervention.

Table 6. Normality of Posttest Scores

Data	Statistic (W)	df	Sig. (p)	Conclusion
Posttest	0.124	30	0.254	Normally distributed

The Kolmogorov–Smirnov test produced a significance value of .254, which exceeded the .05 criterion. This finding indicates that the posttest scores were normally distributed and fulfilled the assumptions required for parametric statistical testing. Since both pretest and posttest datasets satisfied the normality assumption, paired-sample t-test analysis was conducted to determine whether the observed improvement was statistically significant.

Table 7. Paired Sample t-Test Results

Statistic	Value
df	29
p-value	< .001
Mean Difference	37.67
Standard Deviation of Difference Scores	11.420
Standard Error of the Mean Difference	2.085
t	18.063
Cohen's d	3.30

The paired-sample *t*-test revealed a statistically significant difference between the pretest and posttest scores, $t(29) = 18.063$, $p < .001$. The mean difference between the two assessments was 37.67 points (SD of difference scores = 11.420; SE = 2.085), indicating substantial within-group improvement following the instructional intervention. The calculated effect size was extremely large (Cohen's $d = 3.30$), suggesting that the magnitude of improvement was educationally meaningful. Nevertheless, because the study employed a one-group pretest-posttest design without a comparison group, these findings should be interpreted as evidence of statistically significant within-group change rather than definitive evidence that the instructional intervention alone caused the observed improvement.

Table 8. N-Gain Analysis Results

Statistic	Value
N-Gain Minimum	0.45
N-Gain Maximum	0.88
Mean N-Gain	0.64
Category	Moderate

The N-Gain analysis produced a mean score of 0.64, indicating moderate improvement according to Hake's (1998) classification. Individual N-Gain scores ranged from 0.45 to 0.88, suggesting variability in the magnitude of learning improvement across students. Because N-Gain is a descriptive indicator of learning gain rather than an inferential measure of instructional effectiveness, these findings should be interpreted as evidence of moderate within-group improvement following the intervention rather than independent evidence that the instructional approach caused the observed gains.

5. Discussion

The present study examined changes in students' mathematical reasoning following the implementation Problem-Based Learning (PBL) integrated with Deep Learning on junior high school students' mathematical reasoning ability. The findings revealed a statistically significant improvement in students' performance following the intervention, as evidenced by the substantial increase in mean scores from the pretest ($M = 41.73$) to the posttest ($M = 79.40$), the significant paired-sample t-test result ($t(29) = 18.063$, $p < .001$), the extremely large effect size (Cohen's $d = 3.30$), and the medium-to-high N-Gain score ($g = 0.64$). Collectively, these findings indicate that the integration of PBL and Deep Learning was associated with meaningful improvements to the development of students' mathematical reasoning abilities, including understanding problems, constructing mathematical models, applying systematic solution procedures, providing logical justifications, and drawing valid conclusions. The results suggest that learning experiences emphasizing active inquiry, conceptual understanding, and reflective thinking can effectively strengthen mathematical reasoning among junior high school learners.

The observed improvement may be explained by the instructional characteristics of PBL, which position students as active problem solvers rather than passive recipients of information. Throughout the intervention, students engaged with contextual Systems of Linear Equations in Two Variables (SPLDV) problems that required them to analyse situations, identify relevant information, evaluate alternative strategies, and justify their solutions. Such learning experiences encouraged students to construct mathematical understanding through investigation and discussion while simultaneously strengthening analytical and reasoning processes (Nasution, M. S., et al., 2026; Fardian, D., & Dasari, D., 2023). Rather than focusing solely on procedural execution, students were required to explain why particular strategies were appropriate and how mathematical concepts were interconnected. Consequently, reasoning became an integral component of the learning process rather than a secondary outcome.

Although the observed improvements are consistent with the theoretical principles of Problem-Based Learning and Deep Learning, alternative explanations should also be considered. Because the study employed a one-group pretest-posttest design, factors such as repeated exposure to the assessment, normal classroom learning experiences, increased familiarity with the instructional content, maturation, and other contextual influences may have contributed to the observed changes. Consequently, the present findings demonstrate that students showed significant improvement following the instructional intervention but do not establish that the integrated instructional approach alone was responsible for the observed gains. Future studies employing comparison groups or randomized designs are needed to isolate the specific contribution of integrating Problem-Based Learning with Deep Learning.

The findings are consistent with previous studies demonstrating the effectiveness of Problem-Based Learning in promoting higher-order thinking and mathematical reasoning. Glazewski and Hmelo-Silver (2019) reported that PBL enhances students' analytical and argumentative thinking through engagement with complex and authentic problems. Similarly, Savery (2015) emphasized that collaborative problem-solving and reflective discussion within PBL environments help learners develop stronger conceptual connections and reasoning skills. Consistent with these studies, students in the present research were required to examine mathematical relationships, defend their conclusions, and evaluate alternative solution pathways, all of which are essential elements of mathematical reasoning.

Although the present findings provide preliminary support for integrating Deep Learning within Problem-Based Learning, the one-group pretest-posttest design does not permit conclusions regarding whether the integrated approach offers advantages over Problem-Based Learning alone, Deep Learning alone, or conventional instruction. Consequently, the present findings should be interpreted as preliminary evidence supporting the feasibility and potential of the integrated instructional approach rather than evidence of its comparative superiority. Within this integrated framework, Problem-Based Learning provided authentic problem-solving experiences, whereas Deep Learning emphasized meaningful learning, reflection, conceptual understanding, and knowledge integration (Feriyanto & Anjariyah, 2024). These complementary instructional processes are theoretically consistent with improvements in mathematical reasoning; however, future comparative studies are needed to determine the specific contribution of each instructional component.

The findings also support Deep Learning theory, which emphasizes that meaningful understanding emerges when learners actively connect new knowledge with existing cognitive structures. According to Alt and Boniel-Nissim (2018), deeper learning occurs when students engage in elaboration, reflection, and conceptual integration rather than rote memorization. During the intervention, students were encouraged to explain their reasoning, evaluate alternative solutions, and reflect on the effectiveness of their approaches. Such activities promoted deeper cognitive engagement and enabled students to construct more coherent and transferable mathematical knowledge. Consequently, students became increasingly capable of applying mathematical concepts to unfamiliar situations and providing logical justification for their conclusions.

The moderate N-Gain score ($g = 0.64$) obtained in this study indicates that students demonstrated a moderate level of improvement according to Hake's (1998) classification. Because N-Gain is a descriptive measure of improvement, these findings should be interpreted as indicating moderate within-group learning improvement rather than independent evidence of instructional effectiveness. This finding is important because it indicates that the instructional approach may support both achievement improvement and greater equity in learning outcomes. Rather than benefiting only high-performing students, the intervention appears to have provided meaningful learning opportunities for a broader range of learners.

From a practical perspective, the findings suggest that integrating Problem-Based Learning and Deep Learning can support mathematical reasoning through authentic problems, collaborative inquiry, and reflective learning. Such practices align with contemporary goals of mathematics education that emphasize conceptual understanding, critical thinking, and problem solving. Accordingly, the integrated instructional approach represents a promising strategy for mathematics instruction that warrants further evaluation through randomized controlled trials and quasi-experimental studies employing appropriate comparison groups.

Several limitations should be acknowledged. The study employed a one-group pretest-posttest design; therefore, causal conclusions should be interpreted cautiously (Fardian & Dasari, 2023). Because the same mathematical reasoning instrument was administered during both the pretest and posttest, testing effects resulting from prior exposure to the assessment cannot be completely excluded. Furthermore, because Problem-Based Learning and Deep Learning were implemented as an integrated instructional approach, the present study cannot determine the independent contribution of each instructional component to the observed improvement. In addition, the sample was limited to a single class from one junior high school in North Sumatra, which may restrict the generalizability of the findings, and the intervention was conducted over a relatively short period focusing on immediate learning outcomes. Future research should involve larger and more diverse samples, include appropriate comparison groups, examine long-term learning outcomes, and directly compare the integrated approach with Problem-Based Learning alone, Deep Learning alone, and conventional instruction.

Overall, the present study provides preliminary empirical evidence regarding the feasibility and potential of integrating Problem-Based Learning and Deep Learning within an authentic junior high school mathematics classroom. Students demonstrated statistically significant within-group improvement in mathematical reasoning following the instructional intervention; however, the one-group pretest-posttest design does not permit definitive conclusions regarding causality or the comparative advantage of the integrated approach. Nevertheless, the findings contribute to the growing body of research supporting the exploration of instructional approaches that promote meaningful engagement, conceptual understanding, and mathematical reasoning.

6. Conclusion

This study investigated changes in junior high school students' mathematical reasoning following the implementation of Problem-Based Learning integrated with a Deep Learning approach. The findings indicated a statistically significant improvement in students' mathematical reasoning following the intervention, as evidenced by the substantial increase in posttest performance, the significant paired-sample t-test result ($p < .001$), and the moderate N-Gain score ($g = 0.64$). These results indicate that the instructional approach was associated with statistically significant improvements in strengthening students' ability to interpret mathematical problems, formulate mathematical representations, apply systematic solution strategies, provide logical justifications, and draw valid conclusions.

The findings suggest that integrating PBL with Deep Learning creates a learning environment that promotes active inquiry, conceptual understanding, reflective thinking, and meaningful knowledge construction. Through engagement with authentic mathematical problems and structured opportunities for discussion and reflection, students developed stronger reasoning competencies and deeper conceptual understanding. The observed improvement therefore extends beyond achievement gains and reflects meaningful development in students' mathematical thinking processes.

From a practical perspective, the study highlights the potential of PBL integrated with Deep Learning as an instructional strategy for mathematics education, particularly within the context of junior secondary schools. Mathematics teachers may consider incorporating authentic problem-solving activities, collaborative investigation, and reflective learning practices to strengthen students' reasoning abilities and higher-order thinking skills.

Nevertheless, the findings should be interpreted in light of several methodological limitations. First, the study employed a one-group pretest-posttest design; therefore, the observed improvements should be interpreted as evidence of within-group change following the instructional intervention rather than definitive evidence of causality. In addition, the assumption of normality was evaluated using separate pretest and posttest score distributions rather than the distribution of pretest-posttest difference scores recommended for paired-samples t-test analysis. Although both score distributions approximated normality, future research should assess the normality of difference scores to further strengthen statistical rigor. Furthermore, the study involved a relatively small sample drawn from a single junior high school and the intervention was implemented over only four instructional sessions, which may limit the generalizability of the findings to other educational contexts. Future research should employ experimental or quasi-experimental designs with appropriate comparison groups, involve larger and more diverse samples, examine the long-term effects of the intervention, and compare the integrated approach with Problem-Based Learning alone, Deep Learning alone, and conventional instruction. Such investigations would provide a more comprehensive understanding of the potential contribution of integrating Problem-Based Learning and Deep Learning to students' mathematical reasoning development.

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Declaration of Conflicting Interests and Ethics

The authors declare no conflict of interest. This study was conducted in accordance with established ethical principles for educational research involving human participants. Prior to data collection, permission to conduct the study was obtained from the administration of SMP Negeri 1 Sinunukan, Mandailing Natal Regency, North Sumatra, Indonesia. Because the participants were junior high school students, written informed consent was obtained from parents or legal guardians, and student assent was secured before participation. Participants were informed of the purpose of the study, the voluntary nature of their participation, and their right to withdraw at any time without penalty. To protect participants' privacy, no personally identifiable information was collected or reported, and all data were anonymized, kept confidential, and used exclusively for academic research purposes. The authors affirm that the study was conducted with integrity, transparency, and respect for the rights, dignity, and welfare of all participants.

AI Use Statement

The authors used the Claude Opus 5 for language check and proofreading purposes to enhance the comprehensibility of the manuscript. The final version of the manuscript was critically reviewed and approved by the authors in order to maintain the accountability of the study.

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