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ANALYTICAL AND NUMERICAL APPROACH IN THE COURSE MECHANICS FOR TEACHING THE DYNAMICS OF TWO-DIMENSIONAL PARABOLIC MOTION

(Review Article)

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Abstract

This study aims to investigate the effectiveness of integrating analytical and numerical approaches in teaching the concept of two-dimensional parabolic motion with air resistance in a university-level Mechanics course. The study involved 70 third-semester physics students from Negeri Semarang University (UNNES) in 2022. Using a one-shot case study design, students were first introduced to an analytical approach through mathematical derivation of motion equations. Subsequently, a Python-based simulation application was developed employing three numerical methods: Euler, Feynman-Newton, and Runge-Kutta. Students were tasked with analyzing projectile motion trajectories by varying the air resistance coefficient, using tools of their choice—including manual calculations, Excel, or Python. Data were collected through a written test at the end of the course and evaluated based on completeness and depth of conceptual understanding. The results showed that 81.43% of students achieved a *Good*, *Very Good*, or *Excellent* level of understanding, while only 18.57% fell in the *Medium* category. These findings demonstrate that the integration of numerical visualization with analytical methods can significantly enhance students' grasp of abstract physical concepts and support the development of computational skills in physics education.

Keywords: Analytical approach, numerical approach, mechanics, parabolic motion

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1. Introduction

Dynamics of two-dimensional parabolic motion is an important topic of mechanics in physics education. This topic covers the principles of kinematics and dynamics, providing students with basic skills to analyze and predict the behavior of objects under the influence of gravity. Traditional teaching methods for this topic have primarily relied on analytical approaches, which involve solving equations of motion using calculus and algebra. However, these methods, can sometimes be abstract and challenging for students to grasp fully.

In recent years, various teaching methods have been proposed to improve student understanding. Numerical methods, including computational simulations and iterative algorithms, offer an intuitive and interactive way to explore complex physical phenomena. By incorporating these methods, educators can create a more engaging and comprehensive learning experience that caters to diverse student backgrounds and learning styles. Neves et. al (2013) adopted Modellus as a tool in teaching mathematical physics concepts in geosciences to students at introductory levels. Field tests in introductory meteorology courses demonstrated that the use of Modellus helped students overcome difficulties in understanding key physics, scientific computation, and meteorology concepts. Kabigting (2021) evaluated the effectiveness of computer simulation in physics concept learning. The results showed significant difference in scores after the treatment, indicating the impact of the teaching methods on learning outcomes. Ogegbo & Ramnarain (2022) used interactive simulation in teaching electrostatics. Numerical method has also been proposed for other fields beside physics, such as molecular biology (Sun & Zhao, 2023) and control engineering (Vidal Rosas & Fernández, 2022).

The integration of technology into education has been widely adopted across various regions and educational levels. For instance, Çelen (2024) explored the role of information and communication technology (ICT) within the 9th-grade mathematics curriculum in Turkey. The study revealed that ICT is incorporated as one of the three key competencies in the data domain and is primarily used for generating graphs and tables in subtopics such as number theory and algebra. Another relevant example at the secondary education level is provided by Piyatissa (2018), who conducted a field experiment to assess the effectiveness of emphasizing visual representation in physics instruction. The study focused on two core topics in high school physics—kinematics and Newton's laws of motion—by employing computer-assisted learning materials that included diagrams, video clips, animations, and simulations. The findings underscore the potential of deliberately integrating visual elements in classroom instruction to enhance students' understanding and performance in physics.

Several studies have indicated that the comprehension of motion in two dimensions, particularly projectile or bullet motion, poses a significant challenge for students. These challenges include misconceptions such as believing that the direction of a velocity vector of a projectile follows the curved path at every position, misunderstanding the relationship between acceleration and motion direction, and incorrectly assuming that a greater launch angle results in a greater horizontal range (Piten et al., 2017). The difficulties also arises due to the abstract nature of physics concepts that are not easily perceived (Zulkarnain et al., 2023). Study also showed that there are large gender gaps in performance on projectile motion questions, particularly affecting girls and potentially giving them a negative impression of their abilities in physics (Low et al., 2018). To address these challenges, innovative teaching methods such as developing simulations using computer software-based applications (Zulkarnain et al., 2023) and interventions aimed at reducing gender gaps in understanding projectile motion questions have been proposed (Low et al., 2018).

The integration of analytical and numerical approaches in teaching offers several educational advantages. Analytical methods provide a deep theoretical understanding and foster critical thinking skills, while numerical methods enhance students' ability to visualize and experiment with physical concepts in a dynamic and interactive environment. This dual approach not only helps students grasp the fundamental principles more effectively but also prepares them for tackling real-world problems that require both analytical and computational skills. Therefore, in this study we integrated analytical and numerical approaches in teaching the dynamic of two-dimensional motion. Analytical approach was made by formulating mathematical analysis. Meanwhile, for numerical approach, a simulation application was developed using numerical method of Euler, Feyman-Newton and Runge-Kutta to solve the governing equation of the motion. Based on the aforementioned background, the research question was formulated as follows:

How well do students understand the concept of projectile motion with air resistance using both analytical and numerical approaches supported by a custom-developed simulation?

To assess students' understanding, instruction was first delivered using an analytical approach. Following this, the researchers developed a Python-based simulation of projectile motion with air resistance. The simulation was implemented using three numerical methods: Euler, Feynman-Newton, and Runge-Kutta.

2. Theoretical Review

The concept of motion is one of the topics with a very broad scope, both for physics students and physics education students. This broad scope has encouraged researchers to analyze various learning difficulties experienced by students. Nugraheni (2017) analyzed the learning difficulties faced by students in the Mechanics course. The results of this analysis indicated that the primary difficulty lies in mastering basic mathematical skills, particularly differentiation and integration.

Learning media play a crucial role in enhancing students' understanding. Nowadays, learning media are continuously evolving, especially with the integration of interactive media. Ouahi et al. (2022) was identify and discuss the views of Moroccan science teachers on the use and effectiveness of interactive simulations PhET (Physics Education Technology) in student teaching and learning. The results indicate that the use of interactive simulations in investigative science teaching and learning is very effective for both teachers and students, so that they can enhance learning activities and help students to understand scientific concepts effectively. In addition to the development of video-based learning tools (Wahyuni & Sulhadi, 2020), the Matlab programming language has also been applied in modelling and solving building physics problem (Ramos et al., 2005).

The discussion of motion at a basic level typically focuses on one- and two-dimensional cases in ideal condition. Situmeang et al. (2019) designed a self-study module on parabolic motion on an inclined plane without regarding the air resistance. Previously, Kalogiratou et al. (2014) reviewed single step methods of the Runge–Kutta type with special properties and applied them to the two dimensional harmonic oscillator, two body problem, pendulum problem. Borghi (2013) further compared object trajectories in three conditions: no resistance, linear drag force, and quadratic drag force. Borghi's work emphasized the use of the "chain rule" for derivatives, systematically eliminating temporal variables from Newton's laws to derive differential equations in Cartesian trajectory representation.

These studies highlight that two-dimensional motion, particularly projectile motion, poses considerable challenges for students. The complexity increases when air resistance is included, rendering conditions non-ideal and expanding the discussion into three-dimensional space. This example represents a specific case of motion influenced by velocity-dependent forces. If the acting force is expressed solely as a function of velocity v, the differential equations of motion can be formulated in two main forms (Fowles & Cassiday, 2005).

$$F_0 + F(v) = m\frac{dv}{dt} = mv\frac{dv}{dx} \tag{1}$$

where F_0 is the constant force which does not depend on velocity. Both equations are applied according to the specific physical case encountered.

In this work, the study focused on motion under the influence of a velocity-dependent force.

$$\sum\vec{F} = \vec{F}(v) \tag{2}$$

The 2nd Newton's Laws can be written as:

$$\vec{F}(v) = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} \tag{3}$$

so that in this case we obtain

$$t - t_0 = m\int_{v(t_0)}^{v(t)} \frac{d\vec{v}}{\vec{F}(v)} \tag{4}$$

If equation (4) is applied to the case of projectile motion with linear air resistance, the force acting can be expressed as:

$$\sum\vec{F} = \vec{F}(v) = -b\vec{v} \tag{5}$$

where b is the air drag coefficient, a constant whose value can be adjusted based on the magnitude of air resistance. If the discussion begins with the x-axis direction, the drag force can be expressed as: $F_x(v) = -bv_x$

Then,

$$t - t_0 = -\frac{m}{b}\int_{v_x(t_0)}^{v_x(t)} \frac{dv_x}{v_x} = -\frac{m}{b}\, ln\frac{v_x(t)}{v_x(t_0)} \tag{6}$$

Therefore, the velocity can be expressed as

$$v_x(t) = v_x(t_0)\ \exp\left(-\frac{b}{m}(t - t_0)\right) \tag{7}$$

Then, the position can be obtained by integrating the velocity as follow:

$$x(t) - x(t_0) = v_x(t_0)\int_{t_0}^{t} \exp\left(-\frac{b}{m}(t - t_0)\right)dt \tag{8}$$

$$x(t) - x(t_0) = -\frac{m}{b}v_x(t_0)\left\{\left(\exp\left(-\frac{b}{m}(t - t_0)\right)\right) - 1\right\} \tag{9}$$

Therefore, the *x*-position can be expressed as:

$$x(t) = x(t_0) + \frac{m}{b}v_x(t_0)\left(1 - \exp\left(-\frac{b}{m}(t - t_0)\right)\right) \tag{10}$$

The analysis in the y-axis direction is mathematically slightly more complex due to the presence of additional forces besides the drag force, as discussed for the x-axis direction. The forces involved include gravity and drag force, which can be expressed as follows:

$F_y = -bv_y \quad \longleftarrow$ friction force

$F_y = -mg \quad \longleftarrow$ gravitation force

Therefore, the total forces can be expressed as:

$$\sum\vec{F} = \vec{F}(v) = \left(-bv_y - mg\right)\hat{\jmath} \tag{11}$$

The 2nd Newton's law gives

$$m\frac{dv_y}{dt} = -bv_y - mg \tag{12}$$

that can be followed by

$$\frac{dv_y}{v_y + \frac{m}{b}g} = -\frac{b}{m}\ dt \tag{13}$$

so that

$$\ln\frac{v_y(t) + \frac{m}{b}g}{v_y(t_0) + \frac{m}{b}g} = -\frac{b}{m}\ (t - t_0) \tag{14}$$

Therefore, the y axis velocity is defined by

$$v_y(t) = \left\{\left(v_y(t_0) + \frac{m}{b}g\right)\exp\left(-\frac{b}{m}\ (t - t_0)\right)\right\} - \frac{m}{b}g \tag{15}$$

and the y position as function of time is defined by:

$$y(t) = y(t_0) + \frac{m}{b}\left(v_y(t_0) + \frac{m}{b}g\right)\left(1 - \exp\left(-\frac{b}{m}\ (t - t_0)\right)\right) - \frac{m}{b}g(\mathrm{t} - \mathrm{t}_0) \tag{16}$$

Up to here we have obtained the formula for velocity and position in the x and y directions, i.e equation (7), (10), (15) and (16). It can be seen that all the formulas contain exponential term in coefficient b. For small b so that the drag force much smaller than the mass, $b \ll m$, the exponential term can be stated as:

$$\exp\left(-\frac{b}{m}\ (t - t_0)\right) = 1 + \left(-\frac{b}{m}\ (t - t_0)\right) + \frac{1}{2}\left(-\frac{b}{m}\ (t - t_0)\right)^2 + \cdots \tag{17}$$

Therefore, the velocity at x direction in equation (7) can be defined as:

$$v_x(t) = v_x(t_0)\left(1 - \frac{b}{m}\ (t - t_0)\right) \cong v_x(t_0) \tag{18}$$

and the x position in equation (10) can be stated as

$$x(t) \cong x(t_0) + \frac{m}{b}\ v_x(t_0)\ \left(1 - \left(1 - \frac{b}{m}\ (t - t_0)\right)\right) = x(t_0) +\ v_x(t_0)\ (t - t_0) \tag{19}$$

The velocity at y-axis as equation (15) can now be written as

$$v_y(t) \cong \left\{\left(v_y(t_0) + \frac{m}{b}g\right)\left(1 - \frac{b}{m}\ (t - t_0)\right)\right\} - \frac{m}{b}g = v_y(t_0) - g(t - t_0) \tag{20}$$

and the y position as equation (16) can be defined as

$$y(t) \cong y_{(t_0)} + \frac{m}{b}\left(v_y(t_0) + \frac{m}{b}g\right)\left(\frac{b}{m}\ (t - t_0) - \frac{b^2}{2m^2}\ (t - t_0)\ \right) - \frac{m}{b}g(t - t_0) \tag{21}$$

Thus

$$y(t) = y_{(t_0)} + v_y(t_0)(t - t_0) - \frac{1}{2}g(t - t_0)^2 \qquad (22)$$

Therefore, for small b ($b \ll m$), the velocity and position can be determined using simple equations, similar to those in the case without a drag force.

3. Method

This study aims to investigate the effect of air resistance on two-dimensional parabolic motion. The influence of air resistance is analyzed by varying the values of the air drag coefficient. This motion is an extension of the ideal projectile motion model without air resistance. Students are encouraged to understand the concept by performing manual calculations for different drag coefficient values and then visualizing the results in a simplified manner to gain a more comprehensive understanding. As confirmation, the lecturer provides numerical visualizations using various methods, including the Euler method, the Feynman-Newton method, and the Runge-Kutta method.

3.1 Research Design

This study employed a one-shot case study design, which is used to obtain information about the effects of a specific treatment on a single group of respondents. This design was chosen due to its simplicity and suitability for a preliminary study prior to a broader implementation involving a control group. The treatment in this context involved the introduction of a custom-developed simulation by the research team in a Mechanics class, where instruction had traditionally focused heavily on mathematical formulations, often stopping at the derivation of motion equations without deeper exploration of the underlying physical phenomena. At the end of the course session, a test was administered to measure students' level of understanding.

3.2 Participants

This study involved 70 participants drawn from two third-semester physics classes at Universitas Negeri Semarang (UNNES) in 2022. Both classes received the same treatment and were therefore considered homogeneous in terms of instructional exposure.

3.3 Data Collection Tools

The research data were obtained from a written test administered at the end of the course session. The test instrument was designed to be solvable using both manual calculations and numerical methods. Students were presented with a case involving a particle undergoing projectile motion with linear air resistance, where the drag force was proportional to the particle's velocity. The equations of motion in the horizontal and vertical directions, expressed in terms of the position components x and y, were provided, already incorporating the air resistance coefficient b. Students were asked to calculate the particle's position for various values of the air resistance coefficient, generate position-versus-time plots, compare the resulting trajectories for different resistance values, and analyze and draw conclusions about the influence of air resistance on the particle's motion.

3.4 Data Analysis

The collected research data were analyzed in accordance with the study's objective, namely to assess students' conceptual understanding of projectile motion with air resistance. The first set of data concerned the types and distribution of software commonly used by students to perform mathematical computations involving motion equations and to visualize the resulting trajectories graphically over time. The second set of data focused on students' conceptual understanding, which was categorized according to the criteria outlined in Table 1.

Table 1. Criteria for Students' Conceptual Understanding

Score Interval	Criteria
≤ 60	Poor
$61 - 70$	Medium
$71 - 80$	Good
$81 - 90$	Very Good
$91 - 100$	Excellent

The minimum threshold for mastery was set at a score corresponding to a grade of B, which falls within the 71–80 range. Based on this criterion, the intervention in this study is considered effective if more than 60% of the students achieve a conceptual understanding at the *Good* level or higher.

4. Results and Discussion

4.1 Students' Works Using Analytical Method

After the mathematical explanation is provided, students are guided to manually investigate various air resistance coefficients to understand their effects on motion through changes in particle position. Students are tasked with calculating particle positions at specific times using different air resistance coefficients. These calculations can be performed manually with a calculator or using simple applications like Excel. Examples of student work can be seen in Figures 1 and 2.

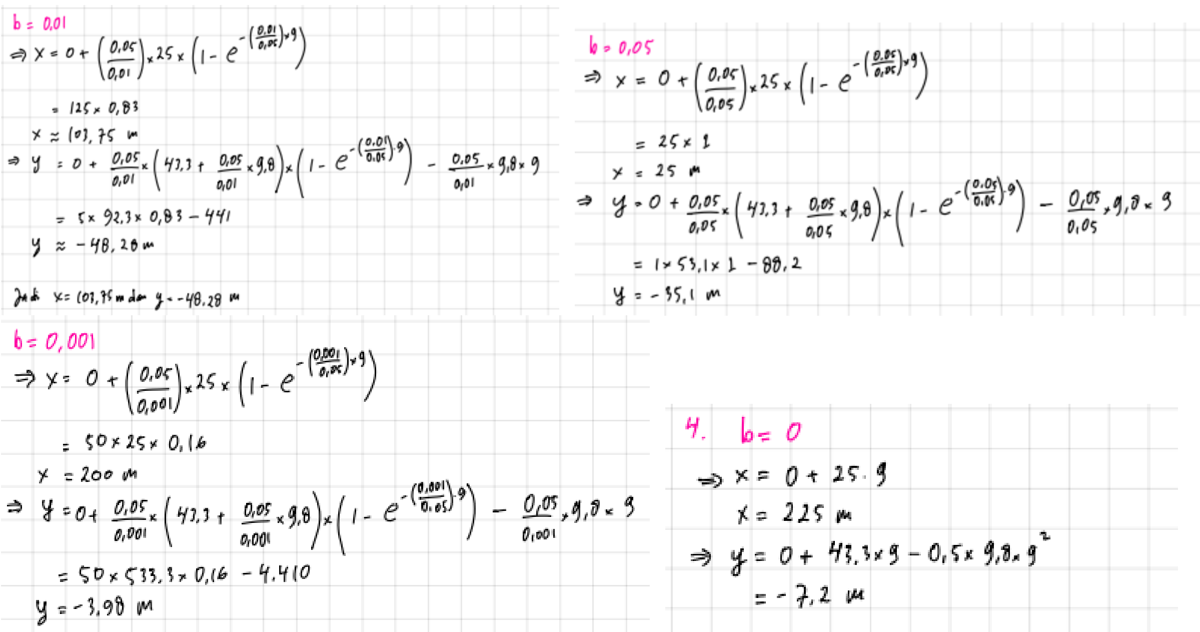

Figure 1: Example of students' works using manual method

untuk b = 0,01

t(s)	x(t)	y(t)	r(t)
1	22,65865588	34,65690873	41,4067145
2	41,20999426	54,1493926	68,0471921
3	56,39854551	61,2262955	83,2433493
4	68,83387951	58,13818044	90,1007824
5	79,01506988	46,72765256	91,7979018
6	87,35072354	28,50330937	91,8835543
7	94,17537953	4,700286053	94,292602
8	99,76293527	-23,67017426	102,532533
9	104,337639	-55,78013569	118,312157
10	108,0830896	-90,9517417	141,259242

Figure 2: Example of student's work using excel.

After obtaining position data for each time interval, students create a simple plot to visualize the trajectory of the object. An example of a simple visualization using Excel is presented in Figure 3. It is evident that the students successfully visualized the trajectory and qualitatively demonstrated that the air resistance coefficient affects the particle's motion.

Figure 3: Student's graphs for various drag forces using excel.

4.2 Development of Simulation Applications

For confirmation, the lecturer also developed a simulation application to help students gain a comprehensive understanding of the material. The computer program chosen for visualization is Python. The part of source code for simulating projectile motion without drag force is provided below.

```
#case of projectile motion without air drag

tl = []
xnl = []
xl = []
ynl = []
yl = []

lerrorx = []
lerrory = []

x=x0
y=y0
v0x = v0 * np.cos(alp)
v0y = v0 * np.sin(alp)
```

```
v = v0
vx = v0x
vy = v0y

b = 0.001
xt = x0 + (m/b)*v0x * (1 - np.exp(-b/m*t))
yt = y0 + (m/b) * (v0y + m/b*g) * (1 - np.exp(-b/m*t)) - (m/b)*g*t

ax = -b/m * vx
ay = -b/m * vy - g

vx = vx + ax * h/2
vy = vy + ay * h/2

while yt >= 0.0:
    tl.append(t)
    xl.append(xt)
    yl.append(yt)
    xnl.append(x)
    ynl.append(y)
    xt = x0 + (m/b*v0x) * (1 - np.exp(-b/m*t))
    yt = y0 + (m/b) * (v0y + m/b*g) * (1 - np.exp(-b/m*t)) - (m/b)*g*t
    x = x + vx * h
    y = y + vy * h
    ax = -b/m * vx
    ay = -b/m * vy - g
    vx = vx + ax * h
    vy = vy + ay * h
    lerrorx.append(round(abs(xt-x),4))
    lerrory.append(round(abs(yt-y),4))
    t += h
plt.plot(xl, yl,'b-')
plt.plot(xnl, ynl,'r-')
plt.axis('equal')
mpld3.display()
```

According to theory, the resulting motion forms a perfect parabola. The visualization of this motion is shown in Figure 4.

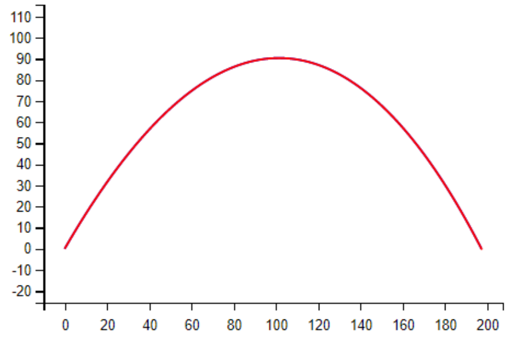

Figure 4: Projectile motion with drag force using simulation

After successfully visualizing the case without air resistance, source code was developed to incorporate frictional forces in the form of air drag. Three numerical methods were employed: the Euler method, the Feynman-Newton method, and the Runge-Kutta method. Each method compared the analytical case without air resistance, the analytical case with air resistance, and the numerical method, all displayed in a single graph. Visualization started with the simplest method, the Euler method. The visualization was achieved by varying the drag coefficient (b), providing an illustration of the effect of air drag on projectile motion. The results of the visualization using the Euler method are presented in Figure 5.

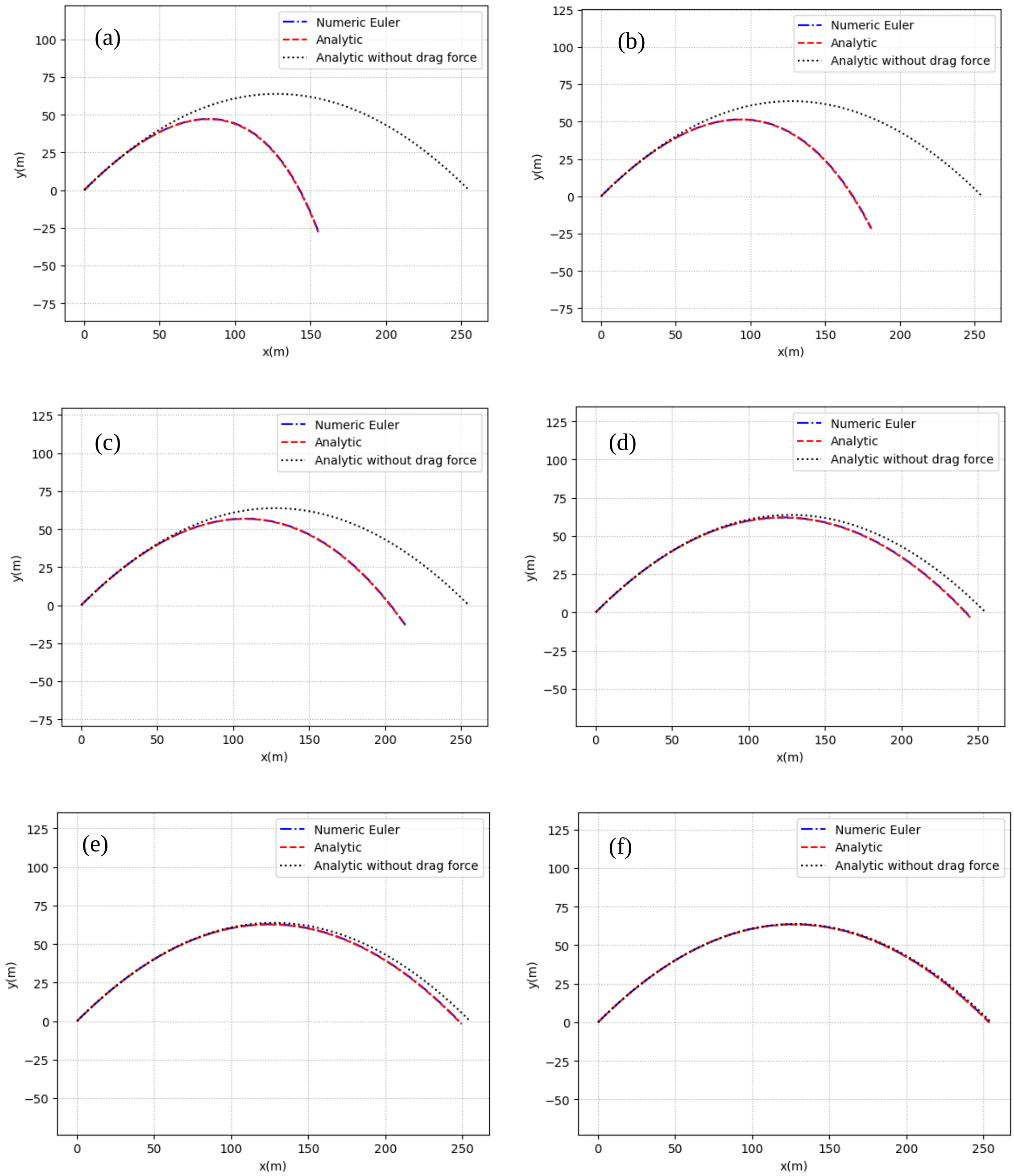

Figure 5: Projectile motion visualization using Euler Method with coefficient of friction of (a) $b = 0.15$ (b) $b = 0.10$ (c) $b = 0.05$ (d) $b = 0.01$ (e) $b = 0.005$ (f) $b = 0.001$

The visualization results obtained using the Euler method were compared with those from the Feynman-Newton method, as presented in Figure 6.

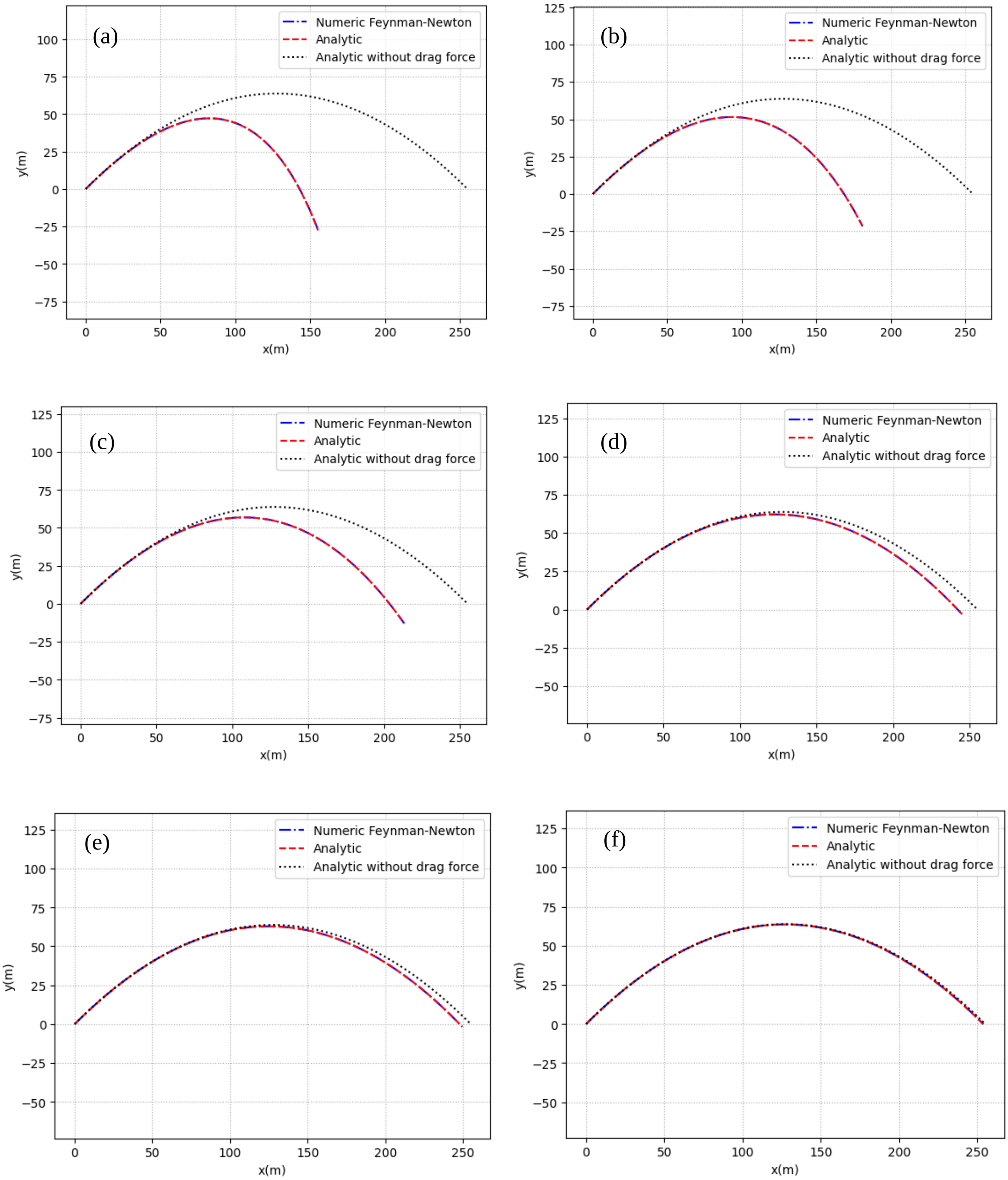

Figure 6: Projectile motion visualization using Feynmann-Newton with coefficient of friction of (a) $b = 0.15$ (b) $b = 0.10$ (c) $b = 0.05$ (d) $b = 0.01$ (e) $b = 0.005$ (f) $b = 0.001$

Simulation was also done for Runge-Kutta method. The results are shown in Figure 7.

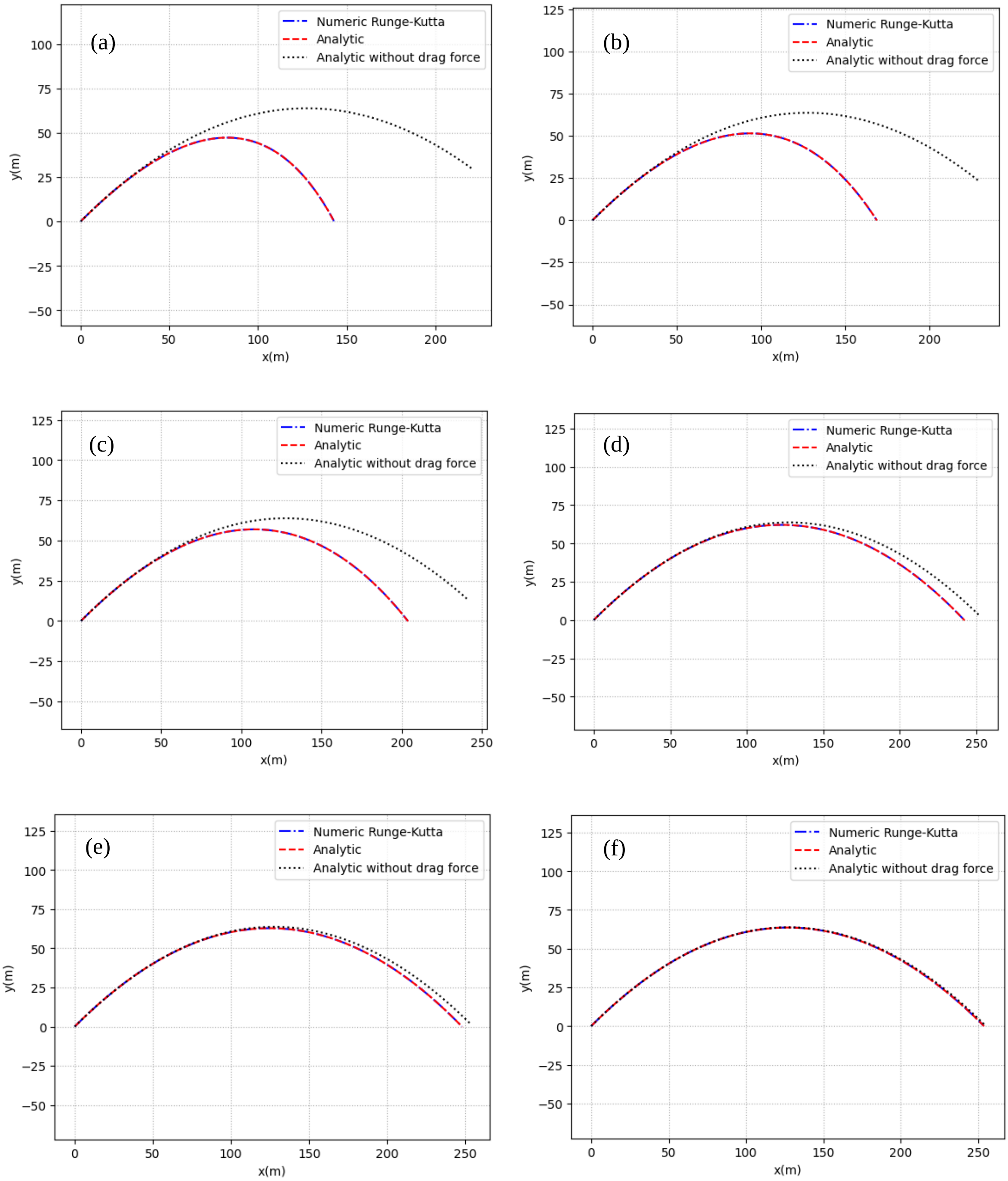

Figure 7: Projectile motion visualization using Runge-Kutta with coefficient of friction of (a) $b = 0.15$ (b) $b = 0.10$ (c) $b = 0.05$ (d) $b = 0.01$ (e) $b = 0.005$ (f) $b = 0.001$

Previously, students were trained to investigate the influence of the drag coefficient manually by calculating changes in the *x* and *y* positions at several time points within a certain interval. The results are shown in Figure 1. Then, Excel was used (Figure 2) to simplify the calculations and visualize the results in graphical form (Figure 3), as manual calculations can be time-consuming, laborious, and tedious. Nevertheless, students need to go through this phase to build a solid understanding. At this stage, students have mastered the concepts of physics, mathematical calculations, and basic visualizations. To further strengthen their understanding, computer-based visualization is introduced through numerical solutions using the Euler method (Figure 5), Feynman-Newton method (Figure 6), and Runge-Kutta method (Figure 7). These three approaches are used to highlight the different options available for solving the problem, as students have already learned the theory in the Computational Physics course. The results from these three numerical approaches demonstrate a consistent understanding of the influence of the drag coefficient on two-dimensional projectile motion. As shown in the three figures, when the value of the drag coefficient (*b*) decreases, the results become closer to the ideal state.

At the end of the course, students were assigned a project to analyze the effect of the air resistance coefficient. The project was conducted with 70 students, who were asked to calculate and compare the trajectories of projectile motion by varying the air resistance coefficient (*b*). Students were free to choose any tools for their analysis, whether manual calculations or specific software. Based on the compiled results, the distribution of software tools used by the students is presented in Figure 8. Although 4.29% of students (3 students) completed the task manually, 24.29% (17 students) utilized Python programming. The majority, 71.43% (50 students), used a simpler tool, which is Microsoft Excel.

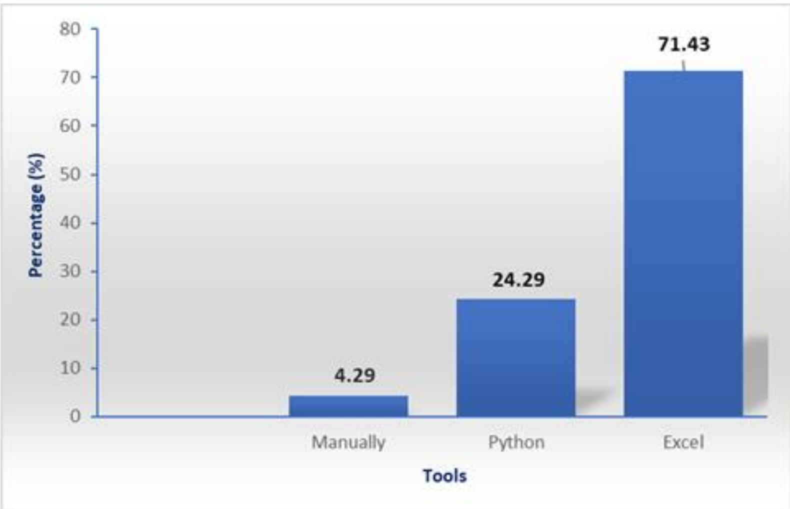

Figure 8: Distribution of students' software choices

The results of the data analysis generally indicate a very high level of students' conceptual understanding, as shown in Figure 9. A dominant proportion of students—81.43% (57 students)—achieved scores within the *Good*, *Very Good*, and *Excellent* categories. This means that only 18.57% (13 students) fell into the *Medium* category.

Figure 9: Students' concept understanding

Students categorized as *Medium* typically submitted incomplete project work, lacking one or more components such as position calculations over time, position-versus-time plotting, and analysis. Among the 13 students in this category, three performed all calculations manually. The *Good* and *Very Good* categories were assigned to students who completed the project in full and were able to generate graphical plots using specific software tools; however, their conclusions lacked clarity or depth. The highest category, *Excellent*, was awarded to students who not only completed all tasks but also demonstrated the ability to analyze the data and draw accurate, well-founded conclusions. An example of a correct conclusion is shown in Figure 10.

> Kesimpulan :
> Dengan melihat nilai numerik dan dari grafik yang ditampilkan maka dapat disimpulkan bahwa semakin kecil gaya hambat udara atau nilai dari b maka semakin jauh jarak tembakan benda bermassa yang dihasilkan dan semakin lama benda jatuh, hal ini dikarenakan semakin kecil gaya hambatnya maka benda akan semakin mudah bergerak dan faktor yang membuat benda jatuh hanya pada pengaruh gravitasi. Pada grafik b = 0 adalah benda bermassa tanpa pengaruh gaya hambat udara jadi jarak tembakannya paling jauh dan dapat digunakan dengan menggunakan rumus gerak lurus berubah beraturan sederhana.

Figure 10: Student answers regarding the investigation of the effect of *b* on projectile motion

It is widely understood that technological literacy plays an important role in supporting students' learning processes. In line with Kaysi's research (2025) which found a positive effect of simulation software on the learning process, this research also brought a positive effect with a more complete understanding of students. This study aligns with the findings of Piyatissa et al. (2018), who emphasized the use of visualization to clarify concepts for students. Their results similarly support the notion that incorporating visualization into classroom teaching can serve as an effective intervention to enhance students' performance in physics. However, the effectiveness of visualization is not without limitations. As noted by Geelan et al. (2012), learning with visualization was found to be no more effective than traditional instruction without visualization for all students, with no statistically significant differences observed across the overall group or across academic ability dimensions. Nevertheless, a greater body of research supports the advantages of visualization in facilitating student understanding, as demonstrated in the studies by Evagorou et al. (2015), Piyatissa et al. (2018), Arista and Kuswanto (2018), and Kaysi (2025).

Manual calculations remain important for training students in logical, step-by-step thinking. However, in the case of projectile motion with air resistance, the resulting mathematical formulations can be lengthy and involve complex terms that many students find challenging. As such, performing all calculations manually can become burdensome. Manual methods are more appropriate for analyzing key points in the motion, such as the launch point, peak height, or range. In this context, visualization becomes essential, as it enables the use of numerical approaches to perform calculations without the need to compute each part step by step. With the correct programming commands, a computer can efficiently carry out the computations and generate graphical plots based on the specified input.

Moreover, not all physical concepts can be readily understood through analytical results alone. Often, these results remain abstract in students' minds. Visualization helps bridge this gap by transforming abstract concepts into concrete representations, making them more accessible and easier to grasp. Even so, a deeper analysis is still necessary to interpret visualization outputs correctly and to ensure that students attain a comprehensive understanding in accordance with theoretical principles.

5. Conclusion

This study demonstrated the effectiveness of integrating analytical and numerical approaches in teaching the dynamics of two-dimensional parabolic motion, particularly in the context of air resistance. Through a one-shot case study involving 70 physics students, the learning intervention combined traditional mathematical analysis with Python-based simulations utilizing the Euler, Feynman-Newton, and Runge-Kutta methods. The results showed that 81.43% of students achieved a conceptual understanding at the Good, Very Good, or Excellent levels, indicating the success of this integrated instructional model.

The inclusion of numerical simulations helped students overcome the abstractness of complex motion equations, enabling them to visualize and better understand the physical behavior of objects under air resistance. While manual calculations remain valuable for fostering structured thinking, the use of software tools like Excel and Python proved instrumental in enhancing comprehension, especially when dealing with velocity-dependent forces.

This dual approach not only deepened students' conceptual understanding but also equipped them with computational skills relevant to physics education. Overall, the findings support the incorporation of numerical methods alongside analytical instruction in Mechanics courses to improve learning outcomes and bridge the gap between theoretical formulation and real-world application.

6. Recommendations

Mathematical courses with an analytical approach, like Mechanics, need to be given a computational touch with a numerical approach so that students gain a more complete understanding of the physics concepts being studied.

7. Ethical Consideration

This educational study did not require formal ethics committee approval according to institutional guidelines, as it involved standard classroom teaching practices and anonymous pre-/post-test assessments with no more than minimal risk. Nevertheless, all participants provided written informed consent before participation.

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